Colloque à la mémoire de

Louis Boutet de Monvel

20-24 juin 2016,
Ecole Normale Supérieure, Paris

Table des matières

1 Programme.

- Lundi 20 juin
  10h30 : Accueil des participants, Salle Jean Jaures, au 29 rue d'Ulm.
  11h Jean-Michel Bony *L'oeuvre mathématique de Louis Boutet de Monvel*
  12h15 : Pause déjeuner
  14h30 : Nalini Anantharaman
  15h30 : Martin Schlichenmaier
  16h30 : pause café
  17h : Bernard Malgrange

- Mardi 21 juin
  9h30 : Philibert Nang
  10h30 : Pause café
  11h : Gerd Grubb
  12h15 : Pause déjeuner
  14h30 : Johannes Sjöstrand
  15h30 : Yves Colin de Verdière
  16h30 : pause café
  17h : Jean-Marc Delort

- Mercredi 22 juin
  9h30 : André Voros
  10h30 : Pause café
  11h : Alain Connes
  12h15 : Pause déjeuner
  14h30 : Philip Boalch
  15h30 : Bernard Helffer
  16h30 : pause café
  17h : Laurent Charles

- Jeudi 23 juin
  Matinée libre
  14h30 : Steve Zelditch
  15h30 : Charles Epstein
  16h30 : pause café
  17h : Jean-Michel Bismut
  à partir de 18h30 : cocktail à l'ENS.

- Vendredi 24 juin
  9h30 : Masaki Kashiwara
  10h30 : Pause café
  11h : Victor Guillemin
  12h15 : Fin du colloque.

2 Titres et résumés.

Nalini Anantharaman, Strasbourg

Dispersion and controllability for linear Schrödinger equations on compact Riemannian manifolds

Abstract : TBA

Jean-Michel Bismut, Orsay

Toeplitz operators and asymptotic torsion

Abstract : Let X be a compact odd dimensional manifold, and let F be a complex flat vector bundle such that $H^{\cdot}(X, F) = 0$. Given metrics on TX and F, analytic torsion is a spectral invariant of the corresponding Hodge Laplacian, that does not depend on the given metrics. If the induced metric on $detF$ is flat, a theorem by Cheeger and Müller asserts that analytic torsion is equal to a combinatorial invariant, the Reidemeister torsion. On locally symmetric spaces, the study of asymptotic torsion was initiated by Bergeron and Venkatesh when X is replaced by coverings of X, and by Müller for high symmetric powers of a flat vector bundle. In the talk, I will describe results on asymptotic torsion I obtained with Ma and Zhang on general manifolds when the vector bundle F tends to $+\infty$ in a suitable sense. Toeplitz operators are used to obtain a lower bound for the Hodge Laplacian, and also in the formulation of the final answer.

Philip Boalch, Orsay

Non-perturbative symplectic manifolds.

Abstract : The wild character varieties are a relatively new class of holomorphic symplectic manifolds first constructed analytically in 1999, although the name is more recent. Their algebraic construction was recently completed in joint work with Daisuke Yamakawa. The simplest example was shown to underlie the Drinfeld-Jimbo quantum group in 2001 and more recent work has explored other examples. A key feature is that in genus zero they have simpler additive analogues, built out of Lie algebras rather than Lie groups. Many well-known symplectic manifolds of classical mechanics occur amongst the additive spaces. The irregular Riemann-Hilbert map plays the role of exponential map, relating the additive and multiplicative symplectic structures. In this talk I will discuss a class of examples related to quivers, Catalan numbers and triangulations, the simplest case of which can be traced back to a short 1979 paper of Louis Boutet de Monvel.

Laurent Charles, IMJ, Paris

Quantum speed limit vs displacement energy.

Abstract : The quantum speed limit is a universal bound on the energy required to pass from one state to another orthogonal state in a quantum system. Similarly, in symplectic topology, the displacement energy is the minimal energy needed to displace a given subset of a symplectic manifold. I will discuss how these two notions are related in the semiclassical limit. Joint work with Leonid Polterovich.

Yves Colin de Verdière, Grenoble

Spectral asymptotics for sub-Riemannian Laplacians.

Abstract : I will present some work with Luc Hillairet and Emmanuel Trélat on spectral properties of sub-Riemannian Laplacians. These operators are hypo-elliptic operators "à la Hörmander" on which Louis contributed a lot. One of our main objective was to get Quantum Ergodicity results for sR Laplacians and more generally to understand the link between their spectral asymptotics and some classical dynamics which in our case is not the geodesic flow. We succeeded for 3D contact distributions. I will also present some concentration results for the eigenfunctions for 3D manifolds with generic singularities of the distribution (Martinet singularities).

Alain Connes, Collège de France

Une équation de Heisenberg pour la géométrie en dimension quatre couplée au modèle standard .

Résumé : J'expliquerai comment le formalisme quantique permet de raffiner la notion Riemannienne d'espace géométrique et d'obtenir la gravitation couplée au modèle standard de la physique des particules à partir d'une version plus élaborée des relations de commutation de Heisenberg.

Jean-Marc Delort, Paris 13

Almost global solutions for the capillarity wave equation with small periodic data.

Abstract : We prove that the capillarity waves equation in one dimension and finite depth has solutions over time intervals of length $c_N\epsilon^{-N}$ for any N, if the Cauchy data are of small size ϵ and space periodic, and if the gravity, or the surface tension, is taken outside a subset of zero measure. The proof relies on normal forms and on the use of the reversibility of the equation. This is joint work with Massimiliano Berti.

Charles Epstein, Philadelphia

Embedding 3d-CR-manifolds, and the sub-elliptic Spin-C Dirac Operators.

Abstract : The problem of deciding whether a perturbation of an embeddable CR-structure on a 3d-contact manifold arises as the boundary of a Stein space, i.e. is "embeddable," dates back to the late 1950s. It took on central importance in Kuranishi's program to construct the versal deformation of an isolated surface singularity. The main difficulty of this problem lies in the fact that the set of embeddable deformations is both infinite dimensional and infinite co-dimensional. In light of this, it is fundamental to show that the set of embeddable deformations is a closed set. This turns out to require a quantitative measure of the stability of the algebra of CR-functions under deformations, which we call the relative index. Using the properties of this relative index, and a very general index formula involving sub-elliptic boundary conditions for a $Spin_C$ Dirac operator, we prove that the set of embeddable deformations is indeed a closed set.

Gerd Grubb, Copenhagen

The transmission property.

Abstract : One of the early mathematical achievements of Louis Boutet de Monvel was to establish in 1966-71 a calculus of boundary value problems for classical pseudodifferential operators P of order m satisfying the so-called transmission condition at the boundary of an open smooth subset Ω of an n-dimensional manifold Ω_1. The condition assures that P preserves $C^\infty(\overline{\Omega})$, and requires m to be integer. At the same time, Hörmander, inspired by works of Vishik and Eskin, proposed a more general μ-transmission condition ($\mu \in \mathbb{C}$) in a photocopy distributed lecture note at IAS Princeton ; it assures that $\operatorname{dist}(x, \partial\Omega)^\mu C^\infty(\overline{\Omega})$ is mapped into $C^\infty(\overline{\Omega})$. The result is included in Hörmander's 1985 book (Section 18.2), but a full discussion of boundary problems for such operators was not given. The talk will explain a theory developed recently (departing from the notes), motivated by the current interest in the fractional Laplacian $(-\Delta)^a$, $0 < a < 1$, which belongs to the case $\mu = a$. I had a good correspondence with Louis about this in the fall of 2013, after we had both touched on the old result in our memorial lectures for Hörmander at the Nordic-European Congress in June 2013.

Victor Guillemin, MIT

Hermite distributions and Toeplitz operators.

Abstract : I will devote the first part of this talk to discussing some of the history (and prehistory) of my collaboration with Louis, and in the remainder of the talk describe various ways in which our results concerning Hermite distributions and Toeplitz operators can be looked at from the "semi-classical" perspective.

Bernard Helffer, Orsay

Magnetic wells in dimension 3 (after Helffer-Kordyukov-Raymond-Vu Ngoc).

Abstract : This talk deals with semiclassical asymptotics of the three- dimensional magnetic Laplacian in presence of magnetic confinement. Using generic assumptions on the geometry of the confinement, we exhibit three semiclassical scales and their corresponding effective quantum Hamiltonians, by means of three microlocal normal forms à la Birkhoff. As a consequence, when the magnetic field admits a unique and non degenerate minimum, we are able to reduce the spectral analysis of the low-lying eigenvalues to a one-dimensional $\hbar$-pseudo-differential operator whose Weyl's symbol admits an asymptotic expansion in powers of $\hbar^2$.

Masaki Kashiwara, Kyoto

Irregular holonomic D-modules and subanalytic sheaves.

Abstract : With Andrea D'Agnolo, I proved the Riemann-Hilbert correspondence. In this talk, I will explain this theory by taking one-dimensional case as an example.

Bernard Malgrange, Grenoble

On the problem of equivalence of Cartan.

Abstract : In a paper of 1910, E. Cartan sketchs a method to decide the local equivalence of two differential structures (at general points). His method has been developped and used by several peoples, including Chern, Gardner, Olver, Kamran. However, there is a problem : Cartan states without proof that, after a certain number of steps, the system becomes involutive (in which case the solution is immediate). The preceding authors found always that this is true in the examples, but no general proof has been given up to now. The main object of this lecture is to give such a proof.

Philibert Nang, ENS Gabon

$\mathcal{D}$-modules on representations of Capelli type.

Abstract : Let (G, V) be an irreducible multiplicity-free finite-dimensional representation of a connected reductive complex group G, and G' its derived subgroup. Denote by $\mathfrak{g}$ the Lie algebra of G, and $U(\mathfrak{g})$ its universal enveloping algebra. Assume that there exists a polynomial f generating the algebra of G'-invariant polynomials on V ($\mathbb{C}[V]^{G'} = \mathbb{C}[f]$) and such that $f \notin \mathbb{C}[V]^G$. Such representations are said to be of Capelli type if the algebra of G-invariant differential operators is the image of the center of $U(\mathfrak{g})$ under the differential of the G- action. They fall into eight cases given by R. Howe and T. Umeda. In this talk we study regular holonomic $\mathcal{D}$-modules associated to these representations.

Martin Schlichenmaier, Luxembourg

Canonical ways to quantize Kähler manifolds.

Abstract : We review some canonical approaches to the quantization of Kähler manifolds (deformation and operator quantizations). A key role will be played by the Berezin-Toeplitz techniques. Beside coherent states, covariant symbols, Berezin transform, an important tool will be given by the generalized Toeplitz operators as introduced by Louis Boutet-de-Monvel and Victor Guillemin.

Johannes Sjöstrand, Dijon

Adiabatic evolution and long time asymptotics.

Abstract : Joint work in progress with A. Mantile and M. Hitrik. Motivated by non-linear mesoscopic Schrödinger evolution problems, we consider the adiabatic problem

$$\big(\varepsilon D_t + P(t)\big)u(t) = 0, (1)$$

where $P(t)$ is a Schrödinger operator with time dependent potential, presenting a potential well in an island. Typically $P(t)$ has shape resonances $\lambda = \lambda(t)$ with$-Im(\lambda) \simeq h^{const}e^{-2S(t)/h}$ where $S(t)$ is the Agmon distance from the well to the sea at energy $Re(\lambda)(t)$. A natural choice in (1) is then $\varepsilon \simeq -Im(\lambda)$. The construction of formal adiabatic solutions is well-known, but for the applications we need to find corresponding exact solutions. Our project is to show that there is an exact solution, matching the formal one on time intervals of length ε^{-N} for arbitrarily large N. One of the main steps was to show that we have a very sharp lower bound on $-Im(P(t))$ when $P(t)$ acts in a space adapted to the resonances, defined by Helffer-Sjöstrand in 1986. (Complex dilations seem insufficient here.) Other steps need some more checking and will be presented as conjectures, proven to $90\%, 95\%$.

André Voros, CEA-Saclay

On simplifying the Keiper/Li approach to the Riemann Hypothesis.

Abstract : We review Keiper and Li's constants $\{\lambda_n\}_{n=1,2,...}$ and their link to the Riemann Hypothesis (RH), a key open question in number theory. The asymptotic ($n \to \infty$) form of that real sequence sharply reflects the truth or not of RH. That leads to conceptually most concrete and practical tests for RH : the prime example is Li's criterion. On the dark side, those numbers have a very elusive nature, and numerically they quickly become intractable. Our analysis, partly semiclassical, makes the Keiper/Li approach more explicit and easier to implement numerically.

Steve Zelditch, Northwestern

Partial Bergman kernels and Quantum Hall.

Abstract : A PBK (partial Bergman kernel) is the orthogonal projection to a subspace $S_k \subset H^0(M, L^k)$ of holomorphic sections of powers of a positive line bundle. Two natural choices of S_k are sections vanishing to high order on a divisor, resp. spectral subspaces of a Toeplitz operator. In each case there is an allowed region, a forbidden region and a transition region between them. We give asymptotics of the PBK in each region. One motivation is to give a rigorous approach to filling a domain with quantum states in the quantum Hall effect. We compare the results to related work of Berman, Ross-Singer, Coman-Marinescu and the physics literature.